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Vortex dominance of the 0^+ and 2^+ glueball mass in $SU(2)$ lattice gauge theory

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Abstract

The c-vortex ensembles are constructed by means of the recently proposed cooling method which gradually removes the $SU(2)/Z_2$ coset fields from the $SU(2)$ lattice configurations and which thus reveals the Z_2 vortex vacuum texture. Using Teper's blocking method, the screening masses of the 0^+ and the 2^+ glueball is calculated from these vortex ensembles and compared with the masses obtained from full configurations. The masses of either case agree within the achieved numerical accuracy of 10%. As a byproduct, we find that the overlaps of the Teper operators with the glueball wavefunctions are significantly larger in the case of the c-vortex ensembles.

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Introduction. The idea that the center part of the $SU(N)$ gauge configurations are important for the confinement of quarks dates back to the late seventies [1]. Indeed, the proposal that the vortex free energy serves as an order parameter for quark (de-)confinement was recently confirmed in the case of a $SU(2)$ gauge group by a large scale lattice simulation [2]. Subsequently, Mach and collaborators explicitly constructed a vortex signature of the Yang-Mills vacuum from gauge invariant variables [3]. It was observed at that time that random fluctuations of the vortex structure disorders the Wilson loop and, hence, provides quark confinement. With the increase of the computer performance in the mid eighties, many research efforts were devoted to substantiate this idea on a quantitative level [4, 5]. One finds that projecting the full lattice configurations onto its vortex content reproduces the string tension of the static quark anti–quark potential. Recently, it was pointed out that not only the asymptotic behavior of the potential but also the short range part which is due to gluon exchange is reproduced if the plaquette is used for a definition of the vortex ensemble [6]. Moreover, the properties of the vortices arising from the plaquette projection technique strongly depend on the size of the lattice spacing thus rendering a continuum interpretation of the latter vortices cumbersome [6].

A significant upturn of the vortex picture of quark confinement occurred with the construction of the p-vortices which are defined after adopting the so-called center gauge [7] by projecting the gauge fixed link variables onto center elements [8, 7]. The fact that Yang-Mills lattice configurations which were reduced to their p-vortex content reproduce the string tension is often referred to as center dominance of the string tension. Moreover, it was observed that the p-vortices are sensible degrees of freedom in the continuum limit [9, 7]: the (area) density of the p-vortices as well as their binary interactions extrapolate to the continuum. The p-vortex picture of the Yang-Mills ground state also provides an appealing explanation of the deconfinement phase transition at finite temperatures [10].

It was pointed out that the center gauge fixing which is prior to identify the physical vortex structure might be plagued by a so-called practical Gribov problem [11]. In addition, it was observed that the vortex properties are quite sensitive to the finite size of the lattice volume [12]. For avoiding the practical Gribov and related problems, a gauge invariant definition of the vortex vacuum texture was achieved by employing a new self-restricted cooling procedure which diminishes the coset fields while leaving the center degrees of freedom un-changed [13]. For rating the phenomenological importance of these, say, c-vortices, center dominance of the string tension was verified. In addition, the $SU(2)$ action density which is carried by the c-vortex vacuum texture properly extrapolates to the continuum limit and, hence, gives rise to a mass dimension four condensate which features in the operator product expansion [13].

In this letter, we will investigate the question whether the c-vortex vacuum texture, besides the appealing picture for quark confinement and their hypothetical signature in high energy hadron collision experiments [13], also provides a quantitative picture of the low lying excitations of pure Yang-Mills theory. For these purposes, we will calculate for the first time the correlation function for the 0^+ and 2^+ glueball channel, respectively, using vortex projected configurations. For this investigation, we confine ourselves to the most simple, but academic case of a pure $SU(2)$ gauge theory. An analogous investigation was recently performed using ablian projection to the maximum ablian gauge [14]: in this case, it was observed that abelian (or even monopole) projected configurations still reproduce the $0^+/2^+$glueball masses known from the full theory.

Glueball correlation functions. Glueball screening masses m_g are calculated from the correlation functions

$$C(t) \;=\; \langle \widetilde{\phi}(t)\, \widetilde{\phi}(0) \rangle \;, \qquad \widetilde{\phi}(t) \;:=\; \phi(t) \,-\, \langle \phi \rangle \tag{1}$$

by analyzing the exponential decrease of $C(t)$ at asymptotic values of t, i.e.

$$C(t) \;\propto\; \exp\Big\{ -m_g t \Big\} \qquad \text{for} \qquad t \gg 1/m_g \;. \tag{2}$$

Thereby $\phi(x)$ is a combination of the link variables which carry the quantum numbers of the glueball under investigation. For choosing a function $\phi(x)$ which generates sufficient overlap with the glueball state, we closely follow the pioneering work of Teper [15] and define composite link variables

$$\begin{aligned} U^{(N)}_i(x) \;&=\; \mathcal{N} \Big\{ U^{(N-1)}_i(x) U^{(N-1)}_i(x+i) \qquad\qquad (3) \\ &+ \sum_{k \neq \pm i} U^{(N-1)}_k(x) U^{(N-1)}_i(x+k) U^{(N-1)}_i(x+k+i) U^{(N-1)\,\dagger}_k(x+i+i) \Big\} \;, \end{aligned}$$

where $i = 1 \ldots 3, k \in -3, -2, -1, 1, 2, 3$ and where $\mathcal{N}$ is a normalization factor to ensure $U^{(N)}_i U^{(N)\,\dagger}_i = 1$. The link variables $U^{(N)}_i(x)$ are defined on a coarser lattice of size $(N^{(N)}_s/2)^3 N_t$ where $N^{(N)}_s$ and N_t are the number of lattice points in the spatial directions and in the time direction, respectively, of the finer lattice. The level $N = 0$ corresponds to the finest level of a $N^{(0)}_s \equiv N_s$. The glueball operators are defined by means of the plaquette $P^{(N)}_{ik}(x)$

$$\phi^{0^+}(t) \;=\; \mathrm{tr} \sum_{\vec{x}} \Big[P^{(N)}_{12}(\vec{x},t) \,+\, P^{(N)}_{23}(\vec{x},t) \,+\, P^{(N)}_{13}(\vec{x},t) \Big] \;, \tag{4}$$

$$\phi^{2^+}(t) \;=\; \mathrm{tr} \sum_{\vec{x}} \Big[P^{(N)}_{12}(\vec{x},t) \,-\, P^{(N)}_{13}(\vec{x},t) \Big] \;, \tag{5}$$

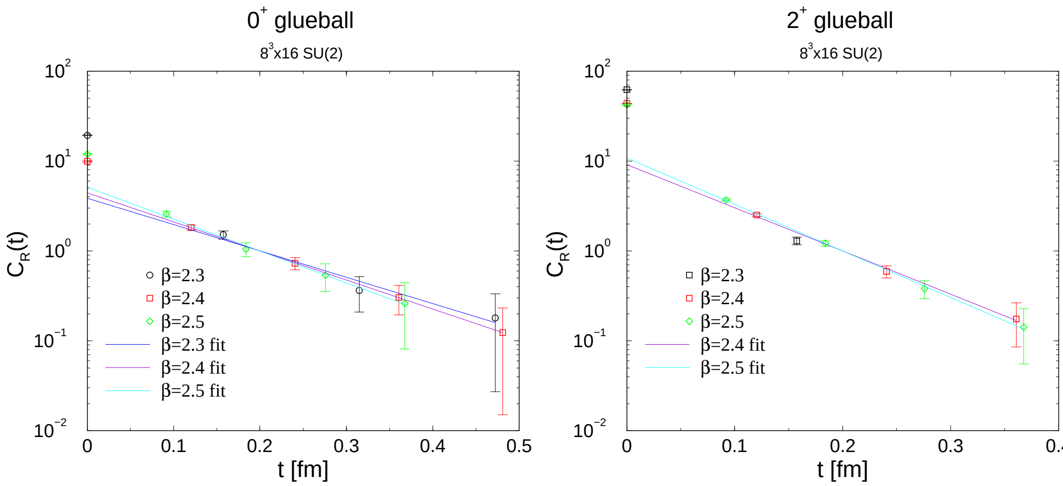

Figure 1: Renormalized correlator $C_R(t)$ (7) in the 0^+ and 2^+ glueball channel, respectively, obtained from full $SU(2)$ configurations.

Here, we use the "blocking" level $N = 2$ throughout the letter. Using the composite link method for the glueball operators, the glueball screening masses were successfully obtained even for the more realistic case of the gauge group $SU(3)$ [16].

In the present paper, we use a $N_s = 8$, $N_t = 16$ lattice and the standard heat bath algorithm to generate the link configurations according to a probability distribution provided by the Wilson action. Using $\beta = 4/g^2$ (where g is the bare gauge coupling) up to values 2.5, we are aware that finite size effects become visible. Our point is, however, to compare the glueball masses calculated from full configurations with those which were obtained by reducing the lattice variables to vortex ensembles rather than to perform a high precision extrapolation to the infinite volume limit. 15000 measurements separated by 10 Monte-Carlo sweeps to reduce auto-correlations were performed to estimate $C(t)$ (1). In order to express t in physical units, we use the β-dependence of the lattice spacing a predicted by one-loop perturbation theory, i.e.

$$\sigma a^2(\beta) \;=\; 0.12 \,\exp\Big\{ -\frac{6\pi^2}{11}(\beta - 2.3) \Big\} \,, \qquad (6)$$

where the string tension $\sigma = (440\text{MeV})^2$ was used as reference scale.

Since $C(0)$ is the expectation value of a composite field, $C(0)$ acquires additional divergencies even if $C(t \neq 0)$ is renormalized finite [17]. Hence, we refrain from normalizing $C(0)$ to 1, but demand

$$C_R(t) \;=\; Z_\phi^2 \, C(t) \,, \qquad C_R(t_0) \;=\; 1 \,, \qquad (7)$$

where t_0 is the renormalization point. From our numerical simulation, the function $C(t)$ at the points $t = a, \ldots, 4a$ obeys the exponential law (2) to good accuracy. This fact allows to evaluate $C(t_0)$ by interpolation where $t_0 = 0.2\,\mathrm{fm}$ is used throughout this paper. The final result $C_R(t)$ is shown in figure 1 for $\beta = 2.3,\ 2.4,\ 2.5$. The corresponding data points are satisfactorily close to a single exponential curve, thus establishing a renormalization group invariant screening mass. Note that $C_R(0)$ changes if different β values are used, thus reflecting the additional divergency associated with the composite operator. The straight lines in figure (1) represent exponential fits to $C_R(t)$ for each β. Averaging over the screening masses obtained by the fit for a given β, we find

$$m_{0^+} \;\approx\; 1.67 \pm 0.11\,\mathrm{GeV}\;, \qquad \mathrm{m}_{2^+} \;\approx\; 2.30 \pm 0.08\,\mathrm{GeV}\;. \tag{8}$$

The uncertainties provided in (8) comprise statistical as well as systematic errors due to the extrapolation to the continuum limit. The masses (8) are consistent with the data presented in [15, 14].

Glueball masses from the c-vortex ensembles. In order to reveal the vortex vacuum structure, we employ the self-restricted cooling procedure proposed in [13] and fractionize the gauge group $SU(2) \hat{=} Z_2 \times SO(3)$. The corresponding degrees of freedom are center vortices and coset fields. The coset part of the $SU(2)$ link variables $U_\mu(x)$ is isomorphic to the adjoint link

$$O^{ab}_\mu(x) \;=\; \frac{1}{2}\mathrm{tr}\Big\{ U_\mu(x)\;\tau^a\; U^\dagger_\mu(x)\;\tau^b \Big\} \;=\; O^{ab}[A^b_\mu]\;, \qquad O^{ab}_\mu(x) \in SO(3)\;, \tag{9}$$

which can be uniquely represented by a gauge vector potential $A_\mu(x)$ in the standard fashion. For removing gluonic (coset) degrees of freedom from $SU(2)$ configurations, the gluonic action density per link is defined by [13]

$$s^{gl}_\mu(x) \;=\; \sum_{\bar\nu \neq \pm\mu} \Big\{ 1 \;-\; \frac{1}{3}\,\mathrm{tr}_A\, O_{\mu\bar\nu}(x) \Big\} \;=\; \frac{1}{3} \sum_{\bar\nu\neq\pm\mu} F^a_{\mu\bar\nu}[A]\; F^a_{\mu\bar\nu}[A]\; a^4 \;+\; \mathcal{O}(a^6)\;, \tag{10}$$

where $O_{\mu\nu}(x)$ is the plaquette calculated in terms of the $SO(3)$ link elements $O_\mu(x)$ (9). The sum over $\bar\nu$ runs from $-4\ldots4$. $F^a_{\mu\nu}[A]$ is the (continuum) field strength functional of the (continuum) gluon fields $A_\mu(x)$.

Cooling is performed by locally reducing the total gluonic action (10) with respect to the fields $O_\mu(x)$. Further cooling of the adjoint link $O_\mu(x)$ is rejected if the gluonic action is smaller than some threshold value

$$s^{gl}_\mu(x) \;<\; 8\kappa^4\, a^4\;. \tag{11}$$

Thereby κ is a gauge invariant cooling scale of mass dimension one. The cooling procedure stops if the gluonic action density (10) locally has dropped below the

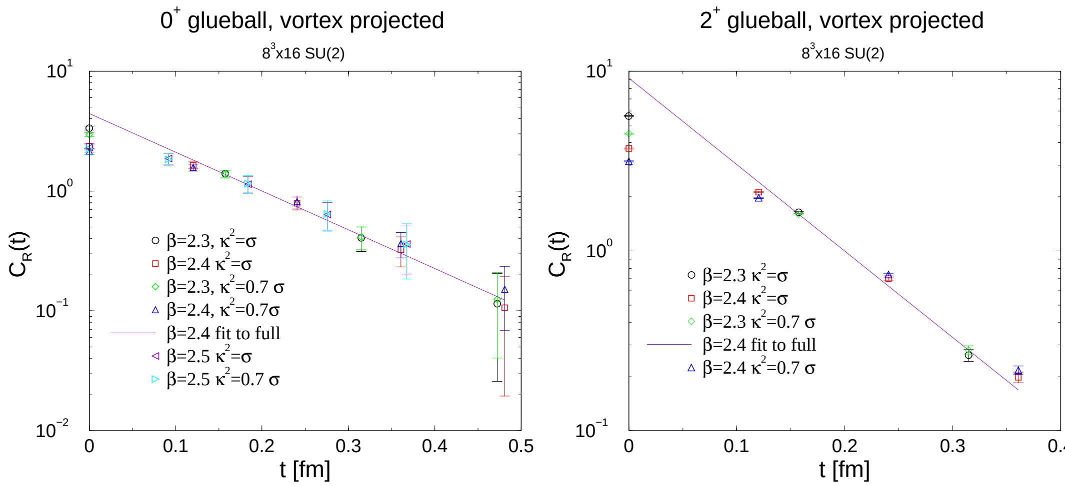

Figure 2: Renormalized correlator $C_R(t)$ (7) in the 0^+ and 2^+ glueball channel, respectively, obtained from c-vortex ensembles.

critical value specified by κ. Details for the practical application of the cooling procedure can be found in [13]. For $\kappa = 0$, the cooling procedure completely removes the gluon fields from the SU(2) lattice configurations leaving only gauge equivalents of $O_\mu(x) = 1$. In fact, even for $\kappa^2 \approx \sigma$ the $SU(2)$ action density is largely clustered along 2-dimensional vortex world sheets, and the short range force between a static quark anti-quark pair is strongly affected. This is expected since the behavior at small distances is dominated by the exchange of gluons, which are already partially eliminated by cooling. Throughout this paper, the configurations which emerge after "gluon" cooling with $\kappa^2 \leq \sigma$ are labeled c-vortex ensembles.

In order to investigate the c-vortex dominance of the glueball screening masses, we evaluate the glueball field combinations (4) and (5) using c-vortex ensembles obtained after adjoint cooling. Note that the action density of the gluon (coset) fields is by construction limited to $\kappa^4 \leq (440\,\text{MeV})^4$ (see (10) and (11)) implying that the large $SU(2)$ action density carried by the c-vortices is required to sustain screening masses of order 1.5 GeV. We employ 5000 measurements to obtained the renormalized correlation function $C_R(t)$ calculated with adjoint cooled configurations, using $\kappa^2 = \sigma$ and $\kappa^2 = 0.5\,\sigma$, respectively. The result is shown in figure 2. Again, the results for several values of β are consistent with a single exponential law (2) reflecting proper scaling towards the continuum limit. The straight line shown in figure 2 is the fit to the data obtained from *full* configurations for $\beta = 2.4$ (see figure 1). We therefore find that the screening masses for the $0^+/2^+$ glueball calculated from full configurations and c-vortex ensembles, respectively, coincide within the achieved numerical accuracy.

Note that the cooling procedure strongly effects the value $C_R(0)$. This is expected since the cooling procedure eliminates UV–divergencies which are generated by gluon (coset) fields of high ($SO(3)$) action density, and therefore alleviates the divergencies of the composite operator. Furthermore, we point out that roughly the same statistical error of the screening masses was achieved in the case of the c-vortex ensembles with a number of measurements which is a factor of three less than the number of measurements employed in the case of full $SU(2)$ configurations. This improvement is due to an enhanced overlap of the "wavefunctions" (4) and (5) with the glueball wavefunctions once the adjoint cooling operates. Following Teper [15] for an estimate of the overlap, we find

$$\left.\frac{C(2a)}{C(0)}\right|_{\text{vortex}} \approx \; 4.6 \left.\frac{C(2a)}{C(0)}\right|_{\text{full}} , \qquad \beta = 2.4,\, \kappa^2 = \sigma \tag{12}$$

for the 0^+ glueball, and

$$\left.\frac{C(2a)}{C(0)}\right|_{\text{vortex}} \approx \; 13.9 \left.\frac{C(2a)}{C(0)}\right|_{\text{full}} , \qquad \beta = 2.4,\, \kappa^2 = \sigma \tag{13}$$

for the 2^+ glueball, respectively.

Conclusions. Recently, a self-restricted cooling method was proposed [13] which gradually removes the $SU(2)/Z_2$ gluon (coset) fields from the $SU(2)$ lattice configurations paving the way to gauge invariant (c-)vortex ensembles. Self-restriction ensures that the local gluonic action density does not exceed the cooling scale κ. By definition, c-vortex configurations are obtained for the choice $\kappa \le \sqrt{\sigma}$, where $\sigma = (440\,\text{MeV})^2$ is the string tension. In the present letter, we have studied the screening masses for the 0^+ and 2^+ glueballs extracted from Teper correlators [15]. Using $\kappa = \sqrt{\sigma}$ and $\kappa = 0.7\sqrt{\sigma}$, we have shown that these screening masses are insensitive to the new cooling method hence providing evidence that the 0^+ and 2^+ glueball masses are dominated by c-vortex configurations.

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